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THE MATHEMATICS OF WARFARE.

HOW TRIGONOMETRY AND CALCULUS ARE CANNED FOR THE GUNNER'S CONVENIENCE.

BY J. MALCOLM BIRD.

The mathematics of warfare is a large subject, with many ramifications; and I shall be able only to identify some of these, and pursue them long enough to indicate the general directions in which they lead. But first I must point out one feature common to almost all war-time mathematics.

Every teacher has at one time or another consoled a student with the remark that while he has plainly made a numerical error, his method is correct. On the field of battle we should have to put it the other end to—that while his method seems correct, he has made a numerical error that renders his result worthless. Perhaps we would also have occasion to observe that he has taken so long to get the answer that even if it were correct the enemy's movements in the meantime would have made it obsolete.

I need not go into argument and example to indicate that the war-time mathematician must usually furnish a prompt answer as well as a correct one. But the field problems of warfare are mainly concerned with the flight of a projectile or the distance and direction of a fixed or moving object. In the one case differential equations are involved, in the other solution of triangles. Now nobody ever lived who could handle differential equations or solve triangles rapidly enough for his results to be of value to a waiting gunner. So most of the mathematics of warfare have to be worked out in advance, and canned for future use in the form of automatic registering instruments and of tables.

I suppose the problems of gunnery are the ones we think of first in connection with the mathematics of warfare. Such problems are peculiarly those of canned mathematics. But in connection with any problem which may come up for solution, it is inferred that certain initial conditions are known; and before showing you through the canning establishment, I propose to say a few words about the instruments used in obtaining these necessary data.

Powder pressure inside the barrel is measured in an experimental gun, drilled with holes at regular intervals. In each hole is set a hard piston backed by a soft metal plug of known compressibility. The compression of these plugs by a given discharge can then be measured and interpreted in terms of powder pressure in pounds per square inch.

Velocities inside the bore are measured in the same perforated gun. The holes are now fitted with cutter plugs, which project into the bore and are sharpened at the other end. When the passing shell forces the plug up its hole, this sharp end cuts a wire, breaking a circuit and making a spark. The spark passes from a tiny metal point to the smoked edge of a rapidly-rotating disk, burning a hole in the sooty coating. A number of the disks are mounted on the same shaft, and the successive plugs connected up so as to mark successive disks. This does away with the confusion that might result from a lot of marks on the same disk. The average velocity of the shell in each interval of the gun is computed directly from the measured distance between the corresponding spark-marks; we have $D/R = d/r$, with everything but r known.

Similar in principle is the chronoscope for measuring velocities during flight, illustrated in Fig. 1. The shell here breaks successive circuits by passing through wire screens. In the first circuit is a magnet M, from which hangs a metal rod R; when the circuit is broken the rod falls. The breaking of the second circuit kicks a little sharp-pointed instrument N forward against the falling rod, nicking it at P. The lag is greater in the second circuit than in the first. This is compensated by placing magnet and nicker in a single trial circuit. When this is broken the rod falls and the nicker jumps; and all measurements are made from the nick Q thus obtained rather than from the projection W of the nicker point upon the rod. The arithmetic of the device is, of course, the same as that of the last.

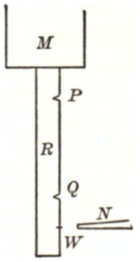

Fig. 1.

Sometimes we want information about the recoil. The very elegant apparatus for giving us this consists of a tuning fork of known period, mounted to vibrate vertically. A stiff bristle attached to this just brushes against a smoked panel fastened to the side of the gun, and of course paints there a complete picture of the recoil—velocity, acceleration, distance covered.

So much for instruments of measurement. Let us pass next to the canning factory where the relations between the velocities, angles, distances and times of flight are worked out and put up in neat packages for the gun officer to take home. We know, of course, that a projectile fired in a vacuum would trace out a parabola. But our gunners have never mastered the art of shooting in a vacuum; and there might be difficulty in getting the enemy to take up his position in a vacuum to be shot at. So we have to practice gunnery under atmospheric conditions.

This introduces a complication. In a well-ordered universe set up for the special convenience of the mathematician, atmospheric resistance to the passage of a solid body would conceivably be constant—constant with respect to the velocity of the body, with respect to size and weight, with respect to any other conditions which we might impose upon the problem. Unfortunately this is is not the case; resistance varies in one way or another with velocity, with size and weight, and with other factors.

Instead of speaking of the net resistance of the air to the passage of the shell, I shall for the present defer to convention and talk about the shell's ability to make head against that resistance. The one, of course, is numerically the reciprocal of the other. This driving power, or figure of merit of the shell, is called the ballistic coefficient and designated by C. Obviously enough C varies directly as the weight of the shell, inversely as the square of its diameter; but this is only a beginning.

There is for one thing the factor of shape. It is clear that a blunt shell will suffer more from the air's resistance than a sharper one. So we have to strike upon a standard shape, assume that κ, the factor of shape, is unity for that shape, and determine its value for other shapes. The standard shell is taken as one with an ogival head struck on a radius of two diameters. The ogive, I may remark, is that type of arch which you must all have recognized as characteristic of two things—Gothic architecture, and the modern projectile. It consists of parallel, rectilinear sides, joined by circular arcs of rather long radius. These arcs are tangent to the straight sides, and of course meet in a point to form the arch. The figure shows the shape of the standard "two-caliber" shell head.

Convention demands that we write κ in the denominator of C, so for better shapes than the standard one it will be less than unity. Thus for an ogival head on a radius of four diameters—a four-caliber head, as it is called—κ is .72; for an eight-caliber head it is 0.5. For an absolutely flat head upon a cylindrical shell κ, on the other hand, would be 2.7, while for a similar head, with rounded shoulders, it would be about 2.

Then there is the factor of steadiness, σ, which likewise goes in the denominator of C. A shell that wabbles badly will present periodically a greater resisting surface to the air than one that travels with normal steadiness. Modern gun construction is so well worked out that for all guns and all shells at direct fire σ is practically unity. But any shell wabbles more at high angle fire than we should like to have it, so σ varies with the initial angle. Thus at angles of 45 degrees or more the trajectory has a very sharp vertex. The shell cannot negotiate this turn with complete success, so for the first part of the descent it sets at an angle of some 20 degrees with its own path, gradually settling down into the correct position, one of approximate tangency to the path. Then, too, we read that the Germans have recently found it necessary to put in temporary use guns with badly worn rifling. Under such firing conditions the shell loses part or even all of its spin, so that it is possible for it to fall blunt end down, or actually to capsize and fall sideways, enormously increasing the value of σ. For the low velocity mortars and howitzers ordinarily employed at high-angle fire a fair value of σ for angles exceeding 30° is 1.2.

The proper value of κ is known and constant for a given gun. In determining σ good deal depends upon local and temporary conditions. The factor of tenuity, τ, which must next be fixed, depends entirely upon such conditions. It is established experimentally that the resistance of the air varies directly as its density. So the ballistic coefficient varies inversely as that density.

Moreover, it is found that excellent results are got by assuming the air's density throughout the flight to be equal to its density at the average height of flight. Because of the lower velocity near the vertex, the average height in a parabolic path would be two thirds that of the vertex; and this assumption of a parabolic path, it turns out, leads to no appreciable error in this connection.

Now under standard conditions—barometer 30 inches, temperature 60° F., humidity 67 per cent.—air weighs 534.22 grains per cubic foot. Tables are furnished giving weight of air for certain combinations of barometer and thermometer readings. The gun officer has to look at these instruments, read from his table, usually by interpolation, the weight of air for the observed conditions at the level of the ground, correct the result for the observed humidity, and divide by 534.22. Then he has to get the tenuity at the mean elevation h of his projectile by means of the formula $\tau_h = \tau_g(1.00003h)$, or from tables compiled from that formula. The result of all this calculation is τ, and since it is smaller than unity when the air is rarer than the standard, it, too, goes in the denominator of C.

It would seem that we have accounted for everything in the world now, and that there is nothing else to think of. This is not quite the case. Some of the formulæ which we are presently to use involve errors with initial angles greater than 10 degrees. An Italian artillerist has worked out a factor of correction for these errors which is a function of the initial angle. This is called, after its originator, Siacci's Beta-function. For angle of fire 11 degrees it is 1.01, and it increases until for 45 degrees it attains the very considerable value 1.26.

We have now

$$C = \frac{w}{d^2 \kappa \sigma \tau \beta};$$

and it R is the total resistance which the air offers to the passage of any shell, and p the resistance to a shell of unit weight and diameter, considered as a function of the velocity alone, we have

$$R = \frac{p}{C}.$$

It is then most natural to ask for an analytic representation of p in terms of v. But, as the old farmer at his first circus said upon beholding the incredible spectacle of the giraffe, "Shucks! They ain't no such animal." All other factors fixed, resistance is indeed a function of velocity, but one that defies all simple analytic representation, one concerning which we can hardly make any definite statement other than that it always increases with v. Its graph (Fig. 2) bears a strong resemblance to the snakes which we have learned from bitter experience to expect from the weaker students in analytic geometry. Among other eccentricities, it has a tremendous kink in the general neighborhood of the velocity of sound, and another at about 2,413 feet per second—the speed at which air rushes to fill a vacuum. There must be something in this double coincidence, but what or why no man knows. Incidentally, this is as good a place

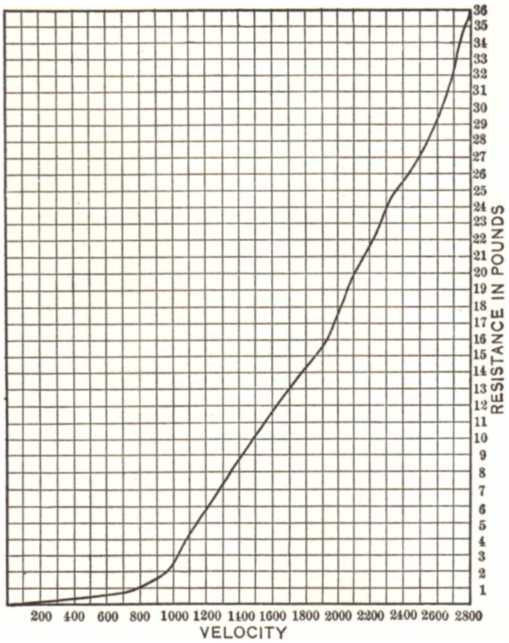

FIG. 2. Curve showing resistance of the air to a projectile 1 inch in diameter.

as any for me to point out that if a shell exceeds this velocity of 2,413 feet per second, it outstrips the efforts of the air to close in behind it, and thus leaves a vacuum in its wake.

Euler, among others, amused himself by deducing what the path of a projectile would be if p varied as the square or the cube of the velocity. Such exercises must now be put in the class of diversions, harmless and useless in equal measure. The determination of the atmospheric resistance, in pounds, to a unit projectile 1 inch in diameter and one pound in weight of the standard shape and under the standard conditions already defined, is wholly empirical. Under these conditions this projectile is fired again and again, and its velocity at various points in its path measured. Comparing the results with those that would obtain if atmospheric resistance were absent, it is easy to deduce the retardation and hence the resistance. In every case, of course, the angle of fire is zero, so that the effect of gravity in diminishing the velocity in the path may be ignored.

The data thus secured have been carefully averaged from countless trials. Together with the differential equations of motion in a resisting medium

$$\frac{d^2x}{dt^2} = -\frac{gp}{C}\cos I; \quad \frac{d^2y}{dt^2} = -\frac{gp}{C}\sin I - g$$

they constitute the raw material from which are constructed the standard ballistic tables that form the chief output of the cannery of war-time mathematics.

These tables are of two sorts—the ballistic tables proper, and the range tables. The former are compiled upon the assumption that we are dealing with direct fire—defined as fire at lower initial angles than 10 degrees. Under these conditions the effect of gravity can be ignored, and the tables constructed almost without reference to the calculus at all.

The first two columns merely set forth the values of p for values of v at intervals of 10 feet per second, as shown upon the graph. Column III. gives ΔT, the time necessary for the projectile to fall from the given velocity to the next one. Since mass × acceleration = force × time, we have, of course, $\Delta T = \Delta v/gp$, where Δv is 10 and p is taken as the mean between the two values concerned. Then Column IV. is merely a summation of the ΔT's to get T, and it works like a definite integral; we find the time it takes the velocity to fall from v_1 to v_2 by subtracting one T-entry from the other. In concise tables for field use the ΔT column is usually omitted, since it consists really of nothing but working figures.

Column V. exhibits $\Delta S = v\Delta T$, the distance traveled while the velocity is dropping Δv feet in ΔT seconds. And of course in column VI. we have S, the summation of the ΔS's: again we subtract the two entries to find the distance traveled in falling from v_1 to v_2, and again we often discard the ΔS column from our finished tables.

The next item involves the effect of g, the gravitational acceleration, in turning the projectile from its initial course. We still work upon the assumption that the initial angle approximates zero; so replacing the cosine by unity and the sine by the angle itself, and making certain other modifications in the differential equations, we get values of ΔD and D, the angle which the axis of the projectile makes with the horizontal plane. This, of course, is the angle I of the differential equations, only here it is measured in degrees, there in radians. As in the previous case we read D by subtraction and discard the ΔD column; and while our results are avowedly derived for low angle fire only, Siacci's Beta-function makes them applicable to all angles.

To complete the ballistic table by the addition of a column showing the height of the projectile above the ground at any point of the trajectory, and to compile range tables which shall exhibit the range for various initial angles and initial velocities, we must fall back upon the differential equations. And here we run into deep water; for in every case we have to deal with an integrand involving both v and p. Now p is a variable, and before integration can be undertaken it must be replaced by its functional value in terms of v. But, as we have seen, this is impossible; so we are at an impassé, and assumptions are in order.

The particular assumptions with which the Gordian knot is cut are rather ingenious. The trajectory is thought of as divided into several partial arcs; and certain of the variables in the intractable integrands are replaced by their mean values over successive arcs. In this way it is possible to plot the trajectory arc by arc, and to control the error by means of Siacci's Beta-function.

The range tables constructed from the results of such computations consist of double entry rows and columns setting forth the range as a function of initial angle and initial velocity. From the ballistic tables and the general range tables it is customary to compile special tables for use with a particular gun of known muzzle velocity. These exhibit, for angles of elevation no more than five minutes apart, not merely range, but velocity and angle of impact, time of flight, penetration into wrought iron, and various other items of less consequence. Even in the absence of such a table, however, it is possible, since the initial velocity is given and the range is known from the general range table, to refer back to the ballistic tables and pick out the remaining velocity of the projectile at the end of its flight, the time of flight and the angle of fall. The problem is merely that of solving for the lower limit when the upper limit and the value of the definite integral are given.

All general tables, of course, are for unit shell at standard conditions. Since the constant C enters linearly in the differential equations it enters linearly in all results, and has to be used as a modifying factor. The presence of the Beta-function in this constant adjusts what would otherwise be serious error. With the special table giving all data for a particular gun, of course, the factor of correction consists only of τ. All the other factors of C are constant or at worst functions of the initial angle which forms the argument of the table itself; and accordingly these can be considered in compiling the table.

With these tables in his hands the worst thing that can come into the life of the gunner is that he be compelled to select the right table and follow a few cross references through its columns, compute the value of C and multiply. Thus if he is shooting at a moving mark, he must know the time of flight in order to allow the enemy the proper time to catch up with the shell fired in front of him. If, as is universally the case with all heavy artillery in the present war, he is using a time shell instead of a contact one, he must again know time of flight, this time to regulate the length of the fuse. If the enemy is using, at known range, a gun of known ballistic properties, his engineers will want to know its remaining velocity and angle of fall so as to estimate its effect upon our works, and our engineers will want to know the same things so as to construct those works at the most advantageous angle to secure deflection, and of a thickness to withstand penetration, if this be possible. If the enemy has been located at B on the far side of a hill, and we must know, without making the trial shot that might reveal our position A, whether a shot ranged at angle θ to hit him will clear the hill, or whether we must fall back to A^1 and fire at a higher angle θ^1, we must ascertain the height of our trajectory at the distance of that hill. A wide variety of such problems the gunner, of course, will solve by means of his tables. But in the daily routine of so pointing his gun that it will hit an enemy in a given position, we can, through the agency of range finder and sight, do even better by him.

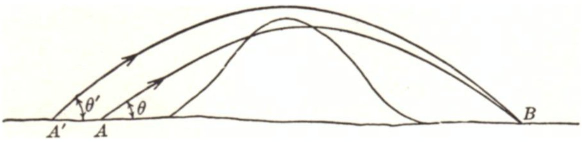

FIG. 3.

A range finder is a machine for automatically solving triangles. We set it according to the known parts, and read off on a scale the unknown part in which we are interested—the distance of the object being ranged. While manufactured in a wide variety of models, mathematically range finders fall into just two classes—the horizontal and the vertical types.

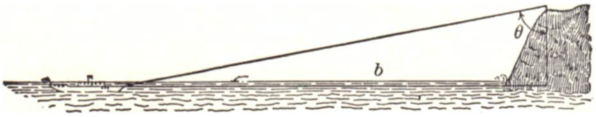

FIG. 4.

The latter is for use in naval work, either in ranging ships from an elevated point on shore, or for use in the crow's nest. Obviously in training a telescope upon a ship from either of these points, a certain angular depression will be necessary.

The telescope, attached to a circular scale, registers this depression automatically; and since the height of the instrument above sea level is known, the triangle is then determined. It is usual to mark the scale with values of tan θ, or if designed for use at a fixed and known height, a, with $a \tan \theta = b$, so that the range can be read directly from the scale.

The horizontal range finder is for more general use. It applies the principle of binocular vision to determine the distance. Our two lines of vision to an object are not parallel; we must look at it cross-eyed; and knowing the distance between our eyes, the angle of ocular adjustment would measure the range.

A layman might object that the contemplated base-line is too small in proportion to the range to give a reliable reading, even with mechanical aid. A mathematician might retort that the astronomer measures star distances of ten light years and more from a base line about seventeen light minutes in length, and that seventeen minutes to ten years is a far smaller ratio than two and one half inches to ten miles. In such an argument the layman, for once, would be right; the ranger has to attain so much greater accuracy than the astronomer that the distance between his eyes is in fact an insufficient base-line for him.

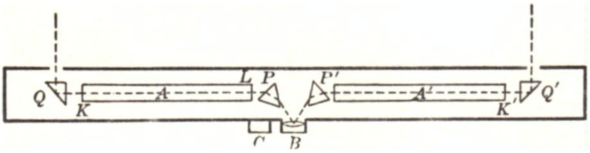

FIG. 5.

So we have to expand this distance; and the horizontal range finder is the result. Two telescopes, A and A^1, are mounted end to end in a carrying tube C, with their objectives K^1 and K in opposite directions. At the eye end L and L^1 of each is a prism P and P^1 which deflects the light passing through both into a common eye-piece B, set in the side of the larger tube. On the opposite side of this tube, at either end, are other prisms Q and Q^1 to deflect into the telescope light falling laterally upon the tube. This light then follows the course of the dotted lines. The observer places his right eye at the common eye-piece of the two telescopes, and of course sees two images of the object being ranged. By means of a thumb screw he turns the prisms through equal angles until these images coincide exactly. In the meantime his left eye is at another eye-piece *C*, in which is visible a scale and pointer; and this registers the amount of rotation that was necessary to bring the axes of the prisms out of their original parallel position into intersection in the object being ranged. This scale, of course, is not marked in angles, but is calibrated according to the distance between the two apertures at Q and Q^1, so that the operator reads off from it, not angles which have then to be converted into ranges, but ranges themselves. In other words, the range finder, in either of its forms, represents twice-canned trigonometry; the results of triangle solution are canned in the ordinary trigonometric tables, and extracts from these tables, by means of which triangles of a given base, a or QQ^1, may be solved, are canned again on the scale of the range finder. The instrument is then fool-proof; anybody with enough eye-sight and intelligence to know when he is seeing double, should be able to operate it after once being shown how. In some models, it should be remarked, the prisms are replaced by mirrors; but mathematically this does not alter the instrument.

The range known, the artillerist turns at once to his sights. At one time it was actually the practice to look in a book to see what angle of fire to use, and then to set the gun at that angle by means of a protractor. Nowadays we take advantage of the fact that a given gun always shoots the same shell with the same charge, and that initial velocity is therefore a function of the gun, and will not change until we change the gun. We simply attach a vertical sight to the gun, and on it we can the portion of the range tables relating to the initial velocity of that particular gun, entering, of course, on each mark, not the angle, but the corresponding range. When we are given the range as 1,760 yards, it is then only necessary to adjust the hind-sight H so that the 1,760 mark M on it is in horizontal alignment with the fore-sight F. In short guns the fore-sight will be at the muzzle; in long bores this would require too long a hind-sight and too tall a gunner, so the fore-sight must be mounted somewhere along the bore, and the hind-sight, which may be straight or curved, graduated accordingly. But these are details. The really significant feature of the elevation sight is that when an enemy is visible, rifle or gun is trained upon him by alining the two sights upon him, just as in short range direct fire—and this of course is equally valid when he is above or below the level of the gun with which we are going to blow him off the map. Then when he is invisible we follow the same procedure, using a spirit level to strike a true horizontal line between *M* and *F*. So the journeyman gunner doesn't have to learn a new procedure for high angle fire at all. The canned mathematics of the range table is canned a second time for his special benefit upon his sight. It must be correctly canned, however; for if the range be wrong, or the sight be wrong, the sighting may be ever so correct and the shell can only plough up the field a few hundred yards in front of the enemy or sail harmlessly over his head.

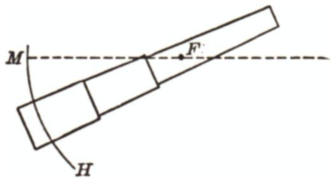

Fig. 6.

When the gun has been properly sighted for range it is still necessary to sight it for direction. Usually the two sights are separate, so that they can be adjusted simultaneously by different men. There can hardly be anything new to the reader in the principle of directing a gun by means of the direction sight; but in its application there enters the interesting factor of drift.

All modern ordnance is rifled. That is to say, the inside of the bore is cut with spiral grooves from end to end, between which project sharp spiral ribs. The body of the shell is of such caliber that it just rests lightly upon these ribs; but the copper rim at the rear has the extreme caliber of the bore, from the bottom of the grooves. As the shell passes up the bore, the sharp, hardened ribs of the rifling cut into this soft copper rim, making grooves in it that have to follow the ribs in their spiral course round and round the inside of the gun. The whole shell consequently acquires an axial rotation as it traverses the bore, and leaves the muzzle with a pronounced spin, which it maintains until it strikes.

This spin adds to the range and makes the flight sufficiently steady to be subject to mathematical laws; it alone renders it possible for the long, thin projectile to remain head on throughout its flight. This it is that makes essential the use of copper for shell rims, a necessity so embarrassing to Germany at the present time. But the spin produced by the rifling also causes the shell to drift to one side, off its course. One is tempted to describe this offhand as analogous to the curve of a pitched ball, or the spin of a sliced golf ball. In reality, it is a different phenomenon, and one not at all well understood. The baseball or golf ball rotates in a horizontal plane containing the line of flight, while the shell rotates in a vertical plane perpendicular to the line of flight. The slice and the curve ball are explained on the ground of greater pressure on the one side of the advancing face of the ball, due to the slipping off of the air in the direction of rotation. In the case of the shell, rotation is not across the advancing face at all, but around it; so another explanation must be sought.

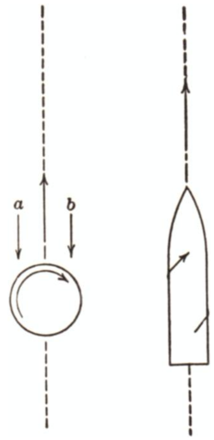

Fig. 7.

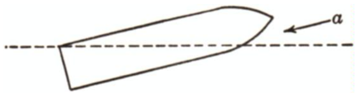

Fig. 8.

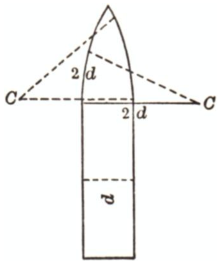

Fig. 9.

About a year ago the German technical papers were devoting a good deal of space to this phenomenon, and the following suggestion was brought forth: The ogival head, in following around after the tangent to the path, as it is forced to do by the inertia of the spin, causes a compression of the air beneath the nose in excess of that existing above. The shell always rides at a slight angle to its own path, and there is a sort of denser air cushion under the advancing head; by virtue of its spin, the shell rolls laterally on this cushion. About that time the German mails ceased to be intermittent and became non-existent, and I have not seen a current German journal in over eight months. Consequently I cannot say how the discussion terminated; but this explanation impressed me as a plausible one. At any rate, the bald fact is that the shell drifts to one side—to the right when shot from a gun with right-handed rifling—and that this deflection increases with the range. Of course, then, after sighting for direction, an arbitrary adjustment of several degrees to the left is necessary.

The possibilities which lie in this complete mathematical control of gun fire reach their climax in the barage fire so extensively employed by our allies. This is a bit of artillery tactics new to the present war. Reduced to its lowest terms, it consists in the training of a large number of guns so that their trajectories all descend along the same straight line. When these guns are then shot at sufficiently short intervals of time, there is a veritable curtain of descending and exploding shells along that line, which cannot be passed by any living thing. The barage may be used to cut the enemy's terrain into two sections, isolated from each other, or to cut him definitely off from a certain portion of our own lines.

Not many years ago this would have been out of the question; the only way to attempt the barage effect would have been to form a single line of identical guns, shooting at the same angle; and it would have been physically out of the question to place them sufficiently close to give an effective barage, to say nothing of the problems of supply, concealment and defense. But today we can distribute guns of all types over an area five or ten miles deep, give each crew the precise range and direction, and go to bed with the secure knowledge that along a given line between us and the enemy, or behind his first defences, there is an impenetrable screen of high explosive shells descending at all angles and all velocities.

It is such artillery co-operation raised to the *n*th power that has made possible the carefully organized operations of our allies in France this spring. An attack is planned several days ahead, and a schedule laid out which must be followed to the instant by all concerned. For perhaps two days beforehand the objective point in the enemy line is isolated from all supply and all relief by a violent barage descending behind it, and at the same time cut to pieces by intense fire directed at it. When the appointed moment comes, the direct fire ceases, and the barage is shifted to a point between the hostile first lines.

Under cover of this curtain the attackers climb leisurely out of their trenches, form in line, and proceed at a walk across No Man's Land. At scheduled time the barage advances; when necessary the advancing line comes to a halt to wait for the barage to move. The whole journey across No Man's Land is thus screened, save for the last dash of from thirty to fifty yards, after the barage has finally been shifted to a point behind the hostile trenches.

Of course the barage does not afford complete protection. In any event losses are severe in the final rush. But it is plain that this scheme of attack, carefully carried out, makes possible sustained offensive without the excessive losses which characterized the similar operations of the previous years of the war. And this scheme rests wholly upon two factors: the absolute mathematical control of the artillery fire, and the absolute mathematical accuracy with which the women munitions workers of England and France make up the shrapnel shells. If the charging infantrymen could not approach to within a few yards of the descending curtain, with confidence that no undercharged shell is going to fall among them and work havoc, these tactics would be out of the question.

I have almost used up my time, and I have spoken of nothing but gunnery. This is really appropriate, because the problems of observation, which formerly shared with those of gunnery the domain of war-time mathematics, have within the past three years been granted a divorce. Observation is no longer a matter of trigonometry; it has become a matter merely of aviation and photography. The whole region behind the enemy's first line is photographed every day and many times a day. These photographs, taken from a known height and with a known lens, are scaled and charted by geometry too elementary to require description. Nothing can take place that is not at once observed by the aviator, reported, and referred to the proper spot on these maps. Then, if it is some activity that should be broken up, a few words by telephone to a gun station, giving merely the indexed location on the map, is sufficient to start the breaking-up process.

The aviator, it is true, has a number of ingenious pieces of apparatus to enable him to determine his altitude, the true vertical direction, the amount of sideways drift, etc.; but I pass over these as being hardly within the field of mathematics. Similarly, I refrain from mentioning the mathematics of war-time engineering, since it is in no essential different from the corresponding technique of peace. I believe I have covered the field of distinctively war-time mathematics sufficiently to give a fair idea of what lies therein, so I will go no further. The reader who is sufficiently interested in this subject to wish to pursue it further will find it thoroughly treated in "Modern Guns and Gunnery," by Col. H. A. Bethell, of the British Field Artillery, and the "British Official Text-Book of Gunnery." The former is published by F. J. Cattermole, of Woolwich, and the latter is an official volume gotten out by the British government.

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